Progress Report

Reference#: N0001492AF00001

PRINCIPAL INVESTIGATOR: Dr. Cha-Mei Tang

INSTITUTION: Naval Research Laboratory

GRANT TITLE: Studies Impacting the Designs of Free Electron Lasers

REPORTING PERIOD: 1/92 - 6/92

AWARD PERIOD: 1/92 - 9/94

OBJECTIVE: 1) To investigate the effect of energy spread, emittance, laser pulse shape and misalignment on x-rays generated by Thomson backscattering using lasers as undulators, and their applications. 2) To address compact FEL issues.

APPROACH: 1) Formulate Thomson, also known as Compton, backscattered radiation for an electron beam with energy spread and finite emittance, laser undulator of arbitrary pulse shape and misalignment. This formulation is to be implemented on the computer. The computer code will be applied to a variety of experimental configurations including Vanderbilt and Duke FEL parameters. We will also evaluate other configurations and medical applications. 2) For FELs to become a widely utilized laser source, they must become more compact, less expensive, and more user friendly. Each design will be strongly dependent on the application. Some common threads are: reducing the beam energy, and reducing the undulator period.

ACCOMPLISHMENTS: (last 6 months): We have formulated the Thomson backscattered radiation for an electron beam with energy spread and finite emittance, misalignment and laser undulator of arbitrary pulse shape (see enclosure 1). We have analyzed the radiation on axis for an idealized step function radiation pulse shape with energy spread and emittance. This simplified analysis is applied to examples where the undulator is a state-of-the-art conventional laser (enclosure 2).

SIGNIFICANCE: The development of a compact, tunable, near monochromatic hard x-ray source would have profound and wide ranging applications in a number of areas, such as x-ray diagnostics, medical imaging, microscopy, nuclear resonance absorption, solid-state physics and material sciences. Using the "K-edge" effect of atoms, the near monochromatic x-rays can significantly enhance the imaging ability of low concentrations of trace elements in the human body by digital differencing the data obtained at two appropriate wavelengths. This can be applied to various cancer detections and digital differential angiography.

The potential of FELs can only be fully achieved when FELs in medicine can be used widely by physicians in hospitals and scientists in universities or research institutes. Thus, we address the issues related to compactness, cost and user interface.

WORK PLAN (next 18 months): The formulation of Thomson backscattering with arbitrary laser pulse shape, misalignment of beams and electron beams with finite energy spread and emittance will be implemented. We will evaluate the x-rays from proposed Vanderbilt Compton backscattering experiments for various electron beam energies. The characteristics of the Mark III electron beam, as well as the laser undulator pulse (the FEL pulse in this case), vary as electron beam energy changes. The Vanderbilt FEL can produce x-rays up to energy about 20 keV. X-rays of 30 KeV range have important applications in angiography. We will also apply this concept to electron beams from accelerators utilizing conventional lasers as an undulator to reach this regime.

Packaging of FELs for a few intended specific applications will be addressed. For medical applications, tunable wavelength in the 1-3 micrometer regime is of particular interest because there is currently an overlap of high transmission of optical fiber and water absorption lines. Laser energy up to 1 J/microsecond would be of interest. We will examine many of the issues effecting FEL design, such as beam quality, cathode, accelerator, undulator, etc.

PUBLICATIONS:

1. "Tunable, Short Pulse Hard X-Rays from a Compact Laser Synchrotron Source", P. Sprangle, A. Ting, E. Esarey and A. Fisher, NRL Memorandum Report 6973; also submitted to the Journal of Applied Physics.

2. "Thomson Backscattered X-Rays from Laser Undulators", Cha-Mei Tang and Bahman Hafizi, submitted to the 14th Intl. FEL Conference, Kobe, Japan, 23-28 Aug 1992. This has been accepted as an invited talk.

3. "Laser Synchrotron Radiation from Beams and Plasmas", E. Esarey, P. Sprangle and S. Ride, submitted to the 14th Intl. FEL Conference, Kobe, Japan, 23-28 Aug 1992. This has been accepted as an invited talk.

4. 1991 FEL Prize talk, "Electron Beam Quality Limitations on Free Electron Laser Operation at High Frequencies", Phillip Sprangle, submitted to the 14th Intl. FEL Conference, Kobe, Japan, 23-28 Aug 1992.

4 August 1992 Final Report 1/92 - 6/92

Progress Report for:

Studies Impacting the Designs of Free Electron Lasers N0001492AF0001

Cha-Mei Tang

Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5320

Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217-5000

Distribution Unlimited

NA

Free electron lasers X-rays 39
Compton backscattering
Thomson backscattering

Unclassified Unclassified Unclassified Unlimited

X-Rays from Laser Undulators

Cha-Mei Tang[1], Bahman Hafizi[2] and Sally Ride[3]

[1] Beam Physics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, DC 20375-5320

[2] Icarus Research, 7113 Exfair Rd., Bethesda, MD 20814

[3] Department of Physics 0319, University of California at San Diego, La Jolla, CA 92093-0021

DRAFT

Enclosure (1)

I. Formulation

We are interested in calculating the Thomson radiation pattern, spectrum and intensity associated with a laser pulse propagating into an electron beam with finite emittance and energy spread going in the opposite direction.

The laser pulse is assumed to be linearly polarized with frequency ω_L. The laser pulse can be separated into fast and slow components,

$$\mathbf{A}(\eta, r) = A(\eta, r)\sin(\eta)\hat{\mathbf{e}}_x, \tag{1}$$

where $\eta = \mathbf{k}_L \cdot \mathbf{r} + \omega_L t$, $|\mathbf{k}_L| = \omega_L/c$. The pulse shape $A(\eta, r)$ and $\mathbf{k}_L(\eta, r)$ are slowly varying functions and $\sin(\eta)$ is a fast oscillating component.

The energy radiated/ unit solid angle/ unit frequency per electron is

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} dt \quad \hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \underline{\beta}) \exp\left[i\omega(t - \hat{\mathbf{n}} \cdot \mathbf{r}/c)\right]\right|^2. \tag{2}$$

For a distribution of electrons with finite emittance and energy spread, the expression becomes

$$\begin{aligned}\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2}\Bigg|\int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathbf{x}_0, \underline{\beta}_0) \\ \int_{-\infty}^{\infty} dt[\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \tilde{\underline{\beta}})] \exp\left[i\omega(t - \hat{\mathbf{n}} \cdot \tilde{\mathbf{r}}/c)\right]\Bigg|^2,\end{aligned} \tag{3}$$

where $f(\mathbf{x}_0, \underline{\beta}_0)$ is the distribution function of electrons in phase space at position $\mathbf{x}_0$ and $\underline{\beta}_0$ at initial time $t = t_0$, $N = \int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathbf{x}_0, \underline{\beta}_0)$ is the total number of electrons, $\underline{\beta} = \mathbf{v}/c$, $\mathbf{v}$ is the velocity, $\underline{\beta}$ is a vector and the symbol "$\sim$" above the variables denotes function of $(\mathbf{x}_0, \underline{\beta}_0, t)$.

A convenient geometric system uses Cartesian coordinates for the velocity and position of the electrons and spherical coordinates for the Thomson backscattering radiation.

$$\begin{aligned}\hat{\mathbf{n}} \times (\hat{\mathbf{n}} \times \tilde{\underline{\beta}}) = &-(\tilde{\beta}_x \cos\theta\cos\phi + \tilde{\beta}_y \cos\theta\sin\phi - \tilde{\beta}_z \sin\theta)\hat{\mathbf{e}}_\theta \\ &+ (\tilde{\beta}_x \sin\phi - \tilde{\beta}_y \cos\phi)\hat{\mathbf{e}}_\phi,\end{aligned} \tag{4}$$

and

$$\hat{\mathbf{n}} \cdot \tilde{\mathbf{r}} = \tilde{x}\sin\theta\cos\phi + \tilde{y}\sin\theta\sin\phi + \tilde{z}\cos\theta, \tag{5}$$

where $\tilde{\beta}_x = \tilde{\underline{\beta}} \cdot \hat{e}_x$, $\tilde{x} = \tilde{\mathbf{r}} \cdot \hat{e}_x$, and similarly for the y and z-components.

We separate the two components of the radiation

$$\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_\theta}{d\omega d\Omega} + \frac{d^2 I_\phi}{d\omega d\Omega}, \tag{6}$$

and perform the calculation in the variable η, where

$$\begin{aligned} \frac{d^2 I_\theta}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2} \Bigg| \int_{-\infty}^{\infty} d^3 x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathbf{x}_0, \underline{\beta}_0) \\ \int_{-\infty}^{\infty} d\eta \left[\frac{\partial \tilde{x}}{\partial \eta} \cos\theta\cos\phi + \frac{\partial \tilde{y}}{\partial \eta} \cos\theta\sin\phi - \frac{\partial \tilde{z}}{\partial \eta} \sin\theta \right] \exp\left[i\tilde{\psi}\right] \Bigg|^2, \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{d^2 I_\phi}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2} \Bigg| \int_{-\infty}^{\infty} d^3 x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathbf{x}_0, \underline{\beta}_0) \\ \int_{-\infty}^{\infty} d\eta \left[\frac{\partial \tilde{x}}{\partial \eta} \sin\phi - \frac{\partial \tilde{y}}{\partial \eta} \cos\phi \right] \exp\left[i\tilde{\psi}\right] \Bigg|^2, \end{aligned} \tag{8}$$

$$\tilde{\psi} = \frac{\omega}{\omega_L}\eta - \frac{\omega}{c}(\tilde{x}\sin\theta\cos\phi + \tilde{y}\sin\theta\sin\phi + \tilde{z}(1+\cos\theta)), \tag{9}$$

and

$$c\tilde{\underline{\beta}} dt = \frac{\partial \tilde{\mathbf{r}}}{\partial \eta} d\eta.$$

Starting at this point, the symbol "$\sim$" on top of the variables denotes function of $(\mathbf{x}_0, \underline{\beta}_0, \eta)$. We assume that the observation point is sufficiently far away that all the electrons can be considered to have the same (θ, ϕ).

For electron with only axial velocity, the electron has two constants of motion in the 1-D limit .[1] In the following, we assume that the electron transverse motion is small, i.e., $(\beta_{x0}^2 + \beta_{y0}^2) << 1$, laser intensity not exceedingly large, i.e., $a << 2\gamma_0/\beta_{z0}$, and $k_\perp \Delta r << 1$, where Δr is the radius of the electron oscillation in the transverse direction driven by the laser. The particle motions are

$$\frac{d\tilde{x}}{d\eta} \simeq \frac{1}{(\omega_L/c)} \left[\tilde{\beta}_{x0} - \frac{\tilde{a}(\eta, r)}{\gamma_0} \sin\eta \right], \tag{10a}$$

$$\frac{d\tilde{y}}{d\eta} \simeq \frac{1}{(\omega_L/c)} \tilde{\beta}_{y0}, \tag{10b}$$

and

$$\frac{d\tilde{z}}{d\eta} \simeq \frac{1}{(\omega_L/c)} \left[\bar{\beta}_1 + \left(\frac{\bar{a}^2(\eta,r)}{4\gamma_0^2} \right) (-1 + \cos(2\eta)) + \bar{\beta}_{x0} \frac{\bar{a}(\eta,r)}{\gamma_0} \sin(\eta) \right], \tag{10c}$$

where $\beta_{z0} = (1 - \gamma_0^{-2} - \beta_{x0}^2 - \beta_{y0}^2)^{1/2}$ is the initial axial velocity, $\bar{\beta}_1 = (1 - (1 + \gamma_0^2(\beta_{x0}^2 + \beta_{y0}^2))/(\gamma_0^2(1+\beta_{z0})^2))/2$, $\bar{\beta}_{x0} = \beta_{x0}/(1+\beta_{z0})$, $\bar{\beta}_{y0} = \beta_{y0}/(1+\beta_{z0})$, $\bar{a}(\eta,r) = a(\eta,r)/(1+\beta_{z0})$ and $a(\eta,r) = (e/m_0c^2)A(\eta,r)$. We assumed that an individual electron does not see the transverse variation of the laser field. Their locations are

$$\tilde{x} = x_0 + \frac{1}{(\omega_L/c)} \left[\bar{\beta}_{x0} \, (\eta - \eta_0) + \int_{\eta_0}^{\eta} \partial\eta' \left(\frac{\bar{a}(\eta',r)}{\gamma_0} \right) \sin\eta' \right], \tag{11a}$$

$$\tilde{y} = y_0 + \frac{1}{(\omega_L/c)} \bar{\beta}_{y0} \, (\eta - \eta_0), \tag{11b}$$

and

$$\begin{aligned} \tilde{z} = z_0 + \frac{1}{(\omega_L/c)} \Bigg[& \bar{\beta}_1 \, (\eta - \eta_0) + \bar{\beta}_{x0} \int_{\eta_0}^{\eta} \partial\eta' \left(\frac{\bar{a}(\eta',r)}{\gamma_0} \right) \sin(\eta') \\ & - \int_{\eta_0}^{\eta} \partial\eta' \left(\frac{\bar{a}^2(\eta',r)}{4\gamma_0^2} \right) (1 - \cos(2\eta')) \Bigg], \end{aligned} \tag{11c}$$

where $\eta_0 = \omega_L(t_0 + z_0/c)$.

The phase can be rewritten as

$$\begin{aligned} \tilde{\psi} = \psi_0 &+ \frac{\omega}{\omega_L} \left[1 - \left(\bar{\beta}_{x0} \sin\theta\cos\phi + \bar{\beta}_{y0} \sin\theta\sin\phi + \bar{\beta}_1 (1 + \cos\theta) \right) \right] (\eta - \eta_0) \\ &- \frac{\omega}{\omega_L} \left(\sin\theta\cos\phi + \bar{\beta}_{x0}(1 + \cos\theta) \right) \int_{\eta_0}^{\eta} d\eta' \frac{\bar{a}(\eta',r)}{\gamma_0} \sin\eta' \\ &+ \frac{\omega}{\omega_L}(1 + \cos\theta) \int_{\eta_0}^{\eta} \frac{\bar{a}^2(\eta',r)}{4\gamma_0^2} d\eta' \\ &- \frac{\omega}{\omega_L}(1 + \cos\theta) \int_{\eta_0}^{\eta} \frac{\bar{a}^2(\eta',r)}{4\gamma_0^2} \cos(2\eta') d\eta', \end{aligned} \tag{12}$$

where

$$\psi_0 = \tilde{\psi}(\mathrm{x}_0, \underline{\beta}_0, \eta_0) = \frac{\omega}{\omega_L}\eta_0 - \frac{\omega}{c} \left(x_0 \sin\theta\cos\phi + y_0 \sin\theta\sin\phi + z_0(1 + \cos\theta) \right).$$

The θ and ϕ components of the radiation are now

$$\begin{aligned} \frac{d^2 I_\theta}{d\omega d\Omega} \simeq \frac{e^2\omega^2c^2}{4\pi^2\omega_L^2} \Bigg| & \int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathrm{x}_0, \underline{\beta}_0) \\ & \int_{-\infty}^{\infty} d\eta \Bigg[\left(\bar{\beta}_{x0} \cos\theta\cos\phi + \bar{\beta}_{y0} \cos\theta\sin\phi - \left(\bar{\beta}_1 - \frac{\bar{a}^2(\eta,r)}{4\gamma_0^2} \right) \sin\theta \right) \\ & - \frac{\bar{a}(\eta,r)}{\gamma_0} (\cos\theta\cos\phi + \bar{\beta}_{x0} \sin\theta) \sin\eta \; - \frac{\bar{a}^2(\eta,r)}{4\gamma_0^2} \sin\theta\cos(2\eta) \Bigg] \exp[i\tilde{\psi}] \Bigg|^2, \end{aligned} \tag{13}$$

$$\frac{d^2 I_\phi}{d\omega d\Omega} \simeq \frac{e^2\omega^2 c^2}{4\pi^2\omega_L^2} \left| \int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathrm{x}_0, \underline{\beta}_0) \right.$$
$$\left. \int_{-\infty}^{\infty} d\eta \left[(\bar{\beta}_{x0}\sin\phi - \bar{\beta}_{y0}\cos\phi) - \frac{\bar{a}(\eta, r)}{\gamma_0} \sin\phi\sin\eta \right] \exp\left[i\tilde{\psi}\right] \right|^2 . \tag{14}$$

At this point we will convert the smooth laser pulse shape into a stepwise continuous function. The laser pulse seen by an electron with initial condition $(\mathrm{x}_0, \underline{\beta}_0)$ is denoted by

$$\bar{\mathrm{a}}(\eta, r) \simeq \sum_{j=1}^{J} \tilde{a}(\eta_j)\sin(\eta)\ h(\eta - \eta_{j-1})\hat{\mathrm{e}}_x, \tag{15}$$

where

$$h(\eta - \eta_{j-1}) = \begin{cases} 1, & \text{for } \eta_{j-1} < \eta < \eta_j \\ 0, & \text{otherwise.} \end{cases}$$

The phase can be integrated giving

$$\tilde{\psi} \simeq \psi_{0,j} + d_0(\eta_j)\eta + d_x(\eta_j)\cos\eta + d_z(\eta_j)\sin(2\eta) \tag{16}$$

for $\eta_{j-1} < \eta < \eta_j$, where

$$d_0(\eta_j) = \frac{\omega}{\omega_L}\left\{1 + \left[-(\bar{\beta}_{x0}\cos\phi + \bar{\beta}_{y0}\sin\phi)\sin\theta - \left(\bar{\beta}_1 - \frac{\tilde{a}^2(\eta_j)}{4\gamma_0^2}\right)(1+\cos\theta)\right]\right\},$$

$$d_x(\eta_j) = \frac{\omega}{\omega_L}\left[\sin\theta\cos\phi + \bar{\beta}_{x0}(1+\cos\theta)\right]\frac{\tilde{a}(\eta_j)}{\gamma_0},$$

$$d_z(\eta_j) = -\frac{\omega}{\omega_L}(1+\cos\theta)\frac{\tilde{a}^2(\eta_j)}{8\gamma_0^2},$$

and

$$\begin{aligned}\psi_{0,j} =& \psi_0 - \left[d_0(\eta_j)\eta_{j-1} + d_x(\eta_j)\cos\eta_{j-1} + d_z(\eta_j)\sin(2\eta_{j-1})\right] \\ &+ \sum_{i=1}^{j-1}\left[d_0(\eta_i)(\eta_i - \eta_{i-1}) + d_x(\eta_i)(\cos\eta_i - \cos\eta_{i-1}) + d_z(\eta_i)(\sin(2\eta_i) - \sin(2\eta_{i-1})\right].\end{aligned}$$

Applying the stepwise continuous radiation pulse shape, it becomes

$$\frac{d^2 I_\theta}{d\omega d\Omega} \simeq \frac{e^2\omega^2 c^2}{4\pi^2\omega_L^2} \left| \int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathrm{x}_0, \underline{\beta}_0) \right.$$
$$\left. \times \sum_{j=1}^{J} \left[g_{0,\theta}(\eta_j) I_0(\eta_j) - \frac{\tilde{a}(\eta_j)}{\gamma_0} I_x(\eta_j)\left(\cos\theta\cos\phi + \bar{\beta}_{x0}\sin\theta\right) - \frac{\tilde{a}(\eta_j)^2}{4\gamma_0^2} I_z(\eta_j)\sin\theta \right] \right|^2 , \tag{17}$$

$$\frac{d^2 I_\phi}{d\omega d\Omega} \simeq \frac{e^2\omega^2 c^2}{4\pi^2\omega_L^2}\left|\int_{-\infty}^{\infty} d^3x_0 \int_{-\infty}^{\infty} d^3\beta_0 \quad f(\mathbf{x}_0, \underline{\beta}_0) \right. \\ \left. \times \sum_{j=1}^{J}\left[g_{0,\phi} I_0(\eta_j) + \frac{\tilde{a}(\eta_j)}{\gamma_0} I_x(\eta_j)\sin\phi\right]\right|^2, \tag{18}$$

where

$$I_0(\eta_j) = \int_{\eta_{j-1}}^{\eta_j} d\eta' \exp\left[i\tilde{\psi}(\mathbf{x}_0, \underline{\beta}_0, \eta')\right], \tag{19a}$$

$$I_x(\eta_j) = \int_{\eta_{j-1}}^{\eta_j} d\eta' \sin\eta' \exp\left[i\tilde{\psi}(\mathbf{x}_0, \underline{\beta}_0, \eta')\right], \tag{19b}$$

$$I_z(\eta_j) = \int_{\eta_{j-1}}^{\eta_j} d\eta' \cos(2\eta') \exp\left[i\tilde{\psi}(\mathbf{x}_0, \underline{\beta}_0, \eta')\right], \tag{19c}$$

$$g_{0,\theta}(\eta_j) = (\bar{\beta}_{x0}\cos\phi + \bar{\beta}_{y0}\sin\phi)\cos\theta - \left(\bar{\beta}_1 - \frac{\tilde{a}^2(\eta_j)}{4\gamma_0^2}\right)\sin\theta,$$

and

$$g_{0,\phi} = \bar{\beta}_{x0}\sin\phi - \bar{\beta}_{y0}\cos\phi.$$

Now, we will integrate over η by expending the exponentials of $\sin\eta$ and $\cos\eta$ into Bessel functions

$$\exp\left[id_z(\eta_j)\sin 2\eta\right] = \sum_{m=-\infty}^{\infty} J_m(d_z(\eta_j))\exp\left[i2m\eta\right], \tag{20}$$

$$\exp\left[id_x(\eta_j)\cos\eta\right] = \sum_{n=-\infty}^{\infty} J_n(d_x(\eta_j))\exp\left[in(\pi/2+\eta)\right] = \sum_{n=-\infty}^{\infty} i^n J_n(d_x(\eta_j))\exp\left[in\eta\right]. \tag{21}$$

Substitute Eqs. (20) and (21) into Eqs. (19a)-(19c), we obtain

$$I_0 = 2e^{i\psi_0}\sum_m J_m(d_z(\eta_j))\sum_\ell i^\ell J_\ell(d_x(\eta_j)) p_{\ell,m}(\eta_j), \tag{22a}$$

$$I_x = -e^{i\psi_0}\sum_m J_m(d_z(\eta_j))\sum_\ell i^\ell [J_{\ell-1}(d_x(\eta_j)) + J_{\ell+1}(d_x(\eta_j))] p_{\ell,m}(\eta_j), \tag{22b}$$

$$I_z = -e^{i\psi_0}\sum_m J_m(d_z(\eta_j))\sum_\ell i^\ell [J_{\ell-2}(d_x(\eta_j)) + J_{\ell+2}(d_x(\eta_j))] p_{\ell,m}(\eta_j), \tag{22c}$$

$$p_{\ell,m}(\eta_j) = \frac{1}{2i(2m+\ell+d_0)}\left(e^{i[(2m+\ell+d_0)\eta_j]} - e^{i[(2m+\ell+d_0)\eta_{j-1}]}\right). \tag{22d}$$

Now, we can evaluate various examples based on this formulation.

II. Fundamental and Harmonics

The emission on axis is peaked at the fundamental and harmonics frequencies, which is embedded in the expression for $p_{\ell,m}$. The expression for $p_{\ell,m}$ can be rewritten as

$$p_{\ell,m}(\eta_j) = \frac{\Delta\eta_j}{2} e^{i[(2m+\ell+d_0(\eta_j))(\eta_j+\eta_{j-1})/2]} \frac{\sin\chi}{\chi},$$

where $\chi = (2m+\ell+d_0)(\Delta\eta_j/2)$ and $\Delta\eta_j = \eta_j - \eta_{j-1}$. Since $p_{\ell,m}$ is peaked around $\chi = 0$. For an electron beam propagating on axis with $\beta_{x0} = 0$ and $\beta_{y0} = 0$, the frequencies of the peak intensity on axis radiated by electron at η_j are

$$\omega_h = h\frac{\omega_L}{1-(\bar{\beta}_1 - (\tilde{a}^2/4\gamma_0^2))(1+\cos\theta)|_{\theta=0}} = h\frac{4\gamma_0^2}{1+a^2(\eta_j)/2}\omega_L, \tag{23}$$

where $h = -2m - \ell$ is the harmonic number and $a(\eta_j) = (1+\beta_{z0})\tilde{a}(\eta_j)$.

The expressions for I_0, I_x and I_z written in terms of the harmonic number become

$$I_0 = 2e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h(\eta_j)\sum_m (-1)^m J_m(d_z(\eta_j))J_{h+2m}(d_x(\eta_j)) \tag{24a}$$

$$I_x = -e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h(\eta_j)\sum_m (-1)^m J_m(d_z(\eta_j))[J_{h+2m-1}(d_x(\eta_j)) + J_{h+2m+1}(d_x(\eta_j))] \tag{24b}$$

$$I_z = -e^{i\psi_0}\sum_{h=1}^{\infty} i^h p_h(\eta_j)\sum_m (-1)^m J_m(d_z(\eta_j))[J_{h+2m-2}(d_x(\eta_j)) + J_{h+2m+2}(d_x(\eta_j))], \tag{24c}$$

$$p_h(\eta_j) = \frac{\Delta\eta_j}{2} e^{i[(d_0(\eta_j)-h)(\eta_j+\eta_{j-1})/2]} \frac{\sin\chi_h(\Delta\eta_j)}{\chi_h(\Delta\eta_j)}, \tag{24d}$$

where $\chi_h = (d_0 - h)(\Delta\eta_j/2)$.

Acknowledgements

The work is supported by the Medical Free Electron Laser Program of ONR.

References

1. E. Esarey, S. Ride and Sprangle, private communications.